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CONSTRAINT ON THE TRILINEAR GAUGE COUPLING BY THE b $\rightarrow$ sγ DECAY CONSIDERING THE POSSIBILITY OF THE FOURTH GENERATION OF QUARKS

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We examine the indirect effect of the new physics. The virtual effects of the trilinear gauge coupling of the photon to W^+W^- on the b $\rightarrow$ sγ decay has been tested considering the possibility of the fourth generation in the quark sector, with corrections up to leading QCD logarithmas, using evolution of the fourth-generation CKM matrix with CP-violation phase equal to zero. Range of mass of the fourth-generation down-type quark b' and that of fourth generation up-type quark t' have been taken with due observance of the constraint imposed by the present experimental value of the ρ parameter, keeping in view the mass difference of the fourth-generation quark doublet. Strong-interaction coupling constant, however, is taken on matching scale of Z-boson mass.

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1. Introduction

It is a fact that due to the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1] in the flavour changing neutral current (FCNC) processes in the Standard Model (SM) [2], the leading order mass-independent term is strongly suppressed by GIM [3] cancellation mechanism, which is experimentally confirmed, and paves the way for investigating the new sources of FCNC. So, the study of virtual effects opened hydraheaded windows

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on electroweak symmetry breaking and physics beyond the SM. The examination of these indirect effects of new physics in higher-order processes yields a complementary approach to the search for direct production of new particles at high-energy colliders.

The new results opened the scope for investigations in various classes of models, namely, anomalous top-quark couplings [4], anomalous trilinear gauge couplings [5], fourth generation [6], two-Higgs-doublet model [7], three-Higgs-doublet model [8], supersymmetry [9], extended technicolour [10], leptoquarks [11] and left-right symmetric models [12].

Recent experimental findings have pointed towards the existence of the fourth generation in the quark sector: down-type quarks d,s,b, and b', and up-type quarks: u,c,t and t' [13]. Several colliders recently endeavour to find some signature for the decay modes like b' $\rightarrow$ bγ and b' $\rightarrow$ bg [14], and so the matter may be dealt with separately or as an extension of the formulation for the three generations to the four generations.

For the third generation at the electroweak scale, the radiative b decay is known to be extremely sensitive to the structure of fundamental interactions. This does not arise at the tree level in the SM, like any other FCNC process. Actually, one-loop W-exchange diagrams generate this decay at the lowest order in SM [15]. The model is based on the gauge group SU(3)$\times$SU(2)$\times$U(1), where the quark couples to the W boson yielding the weak current

$$J^\mu = \frac{g}{2\sqrt{2}} \bar{u}_p \gamma^\mu (1-\gamma_5) V_{pq} d_q \equiv \frac{g}{2\sqrt{2}} V_{pq} J^\mu_{pq}. \tag{1}$$

In the above expression, $p, q = 1, 2, \dots n$ are generation indices and V_{pq} is an $n \times n$ unitary CKM matrix that arises from the quark-mass matrices.

The radiative b decay process is particularly interesting because its rate is of the order $G_F^2\ \alpha_{QED}$, and all other FCNC processes involving leptons and photons are of the order $G_F^2\ \alpha^2_{QED}$. At the same time, the long-range strong interactions are expected to play a minor role in the exclusive process of the radiative B-meson decay (by requiring $E_\gamma > (m_B^2 - m_D^2)/(2m_B)$), and its effect cannot be ignored while considering the inclusive process of radiative b-quark decay. However, it is known that the short-distance effect of QCD [16–18], due to the gluon exchange between the quark lines of the leading one-loop electroweak diagrams, enhance the radiative b-decay rate in the SM by a factor 2 to 5, depending on the top quark mass. Further important contributions in the related domain have been made by Deshpande, Trampetić and others [19–22], where long-distance effects have been duly considered. Other q $\rightarrow$ q'γ transitions are very hard to measure. And recently, the b $\rightarrow$ sγ decay and the top-quark mass have been measured [23,24].

In view of these developments, it is the purpose of the present paper to analyse the theoretical uncertainties in the calculation of the decay width $\Gamma(\mathrm{b} \rightarrow \mathrm{s}\gamma)$. Firstly, this requires the choice of the fourth generation quark masses, which are not free parameters, rather they are constrained by the experimental value of the parameter ρ. The parameter ρ, in terms of the transverse part of the W- and Z-boson self energies at zero momentum transfer, is given by [25]:

$$\rho = \frac{1}{1-\Delta\rho}; \qquad \Delta\rho = \frac{\Pi_{ZZ}(0)}{M_Z^2} - \frac{\Pi_{WW}(0)}{M_W^2}. \tag{2}$$

In the SM, the contribution of a fermion isodoublet (u,d) to $\Delta\rho$ at one-loop order reads:

$$\Delta\rho_0^{SM} = \frac{NG_F}{8\pi^2\sqrt{2}}\, F_0(m_u^2, m_d^2), \tag{3}$$

with the colour factor N, where the function F_0 is given by:

$$F_0(x,y) = x + y - \frac{2xy}{x-y}\, \ln\frac{x}{y}. \tag{4}$$

In the SM, the only relevant contribution is from the top/bottom weak isodoublet. Because $m_t \gg m_b$ yields $\Delta\rho_0^{SM} = (3G_f m_t^2)/(8\pi^2\sqrt{2})$, and since the mass of the top quark is now known to be very high, the fourth-generation quark mass doublet is left with the constraint $\rho(\mathrm{t}' - \mathrm{b}') \le 0.002$ [26] by the experimental data. Adopting $m_b = 4.6$ GeV and $m_t = 160$ GeV (i.e., $m_t(\mathrm{pole}) = 169$ GeV) for $\tan\beta = 1.5$ the allowed region is extended up to $m_{b'}, m_{t'} \le 120 - 125$ GeV, for $\tan\beta = 2.2$ (and higher), larger $m_{t'}$ values but smaller or equal than m_t are possible, but only for $m_{b'} < M_Z$. The constraint on $m_{b'}$ depends greatly upon the manner in which it decays. If $m_{b'}$ has significant mixing with the second or first generation, then b' $\to$ (c, u)+W (where W may be real or virtual) decay will be dominant. and $m_{b'} > 85$ GeV at the 98% C.L. However, if unmixed with lower generations, the b' will decay via FCNC channels: b $\to$ sγ , bg or bZ* for $m_{b'} < M_Z$, and with b' $\to$ bZ becoming dominant for $m_{b'} > M_Z$.

Holdom [27] remarked that the CP violation, originating in the right-handed neutrino sector, can feed into the quark sector, in an otherwise CP-invariant theory. The dominant effects are superweak, and it builds on and extends a previously proposed model of quark masses based on a new strong flavour interaction above the weak scale, and this yields the (t', b') masses close to a TeV. The quark and the charged-lepton masses may be described in terms of operators of the LRLR form, and the value of the up-type masses as (0.002, 74, 160, 1000) GeV, and the down-type masses as (0.005, 0.1, 3, 1000) GeV are expected to be approximate for the masses renormalized at a TeV, and that a fourth family with these dynamical masses can still be consistent with the precision electroweak measurements.

However, Grounau and London [28] are of the opinion that there is a model-independent lower bound of 45 GeV on the mass of t' coming from LEP. There are stronger constraints on the $m_{t'}$ of O(100) GeV, coming from hadron colliders, but this can be evaded since they depend on how strongly t' couples with b-quark. There is an upper bound of 550 GeV on $m_{t'}$ coming from partial-wave unitarity [29]. A heavier t' will lead to a breakdown of the perturbation theory.

However, the model-independent lower bound of the b'-quark mass has been set as 45 GeV [30]. For the present paper, the value of $m_{t'}$ has been kept less than 550 GeV, and m_b has been kept 45 GeV and above, and the pair of values of $(m_{b'}, m_{t'})$ were chosen keeping due respect to $\rho(\mathrm{t}' - \mathrm{b}') \le 0.002$, and one can take the pair of masses as (45,110), (45,115), (50,110), (50,115), (50,120), (60,110), (60,115), (60,120), (60,130), a set with $m_{b'} < M_Z$, and for $m_{b'} > M_Z$, (85,110), (85,115), (85,120), (85,130), (85,140), (85,150), (90,110), (90,120), (90,130), (90,140), (90,150), for $m_{b'} \gg M_W$. Also, for $m_{t'} \gg m_t$ but less than 550 GeV, the following set was assumed: (440,500), (450,500), (460,500), (470,500), (480,540), (490,540), (500,540).

Secondly, the dominant uncertainty in the existing leading logarithmic calculations is due to the choice of the renormalization scale μ [31]. Such an uncertainty, inherent in any finite-order perturbation theory, has recently been analysed in several papers [32–34]. Also, it has been demonstrated in several papers that the inclusion of next-to-leading-order corrections reduces considerably the μ dependence of the relevant amplitudes. The radiative decay of the B-meson, which is formed of b-quark, is dominated by QCD effects. It is not unnatural that the scale uncertainty in the leading-order calculation of this decay is particularly large, it amounts to around $\pm$ 25%. Therefore, one finds restrictions on the SM or its extensions which can be obtained with the help of experimental findings and the leading-order approximation, which are substantially weaker than found without taking the theoretical uncertainties into account.

The μ-dependence in the branching ratios can be reduced in the same manner as was done for other decays [32–34]. The full next-to-leading log calculation of radiative decays would require consideration of three-loop mixing between certain effective operators, before one undertakes such an effort. It is much better to make a formal analysis of the considered decay at the next-to-leading level and to check to what extent the μ-dependence can be reduced once all the necessary calculations have been performed. A review paper by Greub et al. has appeared at a recent symposium [35] on the next-to-leading logarithmic results for FCNC of radiative b decays. Chetyrkin et al. [36] have obtained the results for three-loop anomalous dimensions while analysing $\mathrm{B} \to \mathrm{X}_s\gamma$ decay, and report the branching ratio $Br(\mathrm{B} \to \mathrm{X}_s\gamma) = (3.28 \pm 0.33) \cdot 10^{-4}$. The predictions of the SM are in conformity with the CLEO data at the 2σ level.

However, for the present, the analysis of $\mathrm{b} \to \mathrm{s}\gamma$ decay is restricted to leading logarithmic calculations taking into account the trilinear gauge coupling of photon to $\mathrm{W}^+\mathrm{W}^-$, considering the possibility of the fourth generation in the quark sector. We endeavour to find the constraints on the parameters, taking the CLEO data [23] on $\mathrm{b} \to \mathrm{s}\gamma$ decay as

$$10^{-4} < Br(\mathrm{b} \to \mathrm{s}\gamma) < 4 \times 10^{-4}.$$

The paper is organized as follows: Section 2 summarises the formulation of $\mathrm{b} \to \mathrm{s}\gamma$ decay up to the leading logarithmic (LL) calculations, adopting operator product expansion and incorporation of the trilinear gauge, and also the effect of the fourth generations has been taken into account. In Section 3, results are discussed, and the method of calculation of fourth-generation CKM mixing matrices from quark masses is given in the Appendix.

2. Theory

This section briefly summarises how the leading logarithmic calculation of $\mathrm{b} \to \mathrm{s}\gamma$ decay may be applied to the trilinear gauge coupling of photon to $\mathrm{W}^+\mathrm{W}^-$.

The process $\mathrm{b} \to \mathrm{s}\gamma$ may be taken via W-boson and u-, c-, t-quark exchange in a one-loop penguin-type diagram and for this the factorization of the effective Hamiltonian may be written in the form

$$\mathcal{H}^{eff} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu) O_i(\mu). \tag{5}$$

The QCD corrections to the b $\rightarrow$ sγ decay contain large logarithms $\ln(M_W^2/m_b^2)$ which have to be resummed with the renormalization group equation (RGE). In order to do this, one has to introduce an effective Hamiltonian, built out of operators of dimension higher than four. Here V_{ij} are the CKM matrix elements, $O_i(\mu)$ are the relevant operators and $C_i(\mu)$ are the corresponding Wilson coefficients in the mass scale μ. The complete set of operators necessary in the b $\rightarrow$ sγ, after the t-quark and W-boson have been integrated out, may be taken from Ref. 37.

With this Hamiltonian, one obtains the partial decay width for b $\rightarrow$ sγ as

$$\Gamma(\mathrm{b} \rightarrow \mathrm{s}\gamma) = \frac{\alpha G_F^2 m_b^5}{128\pi^4} |V_{ts}^* V_{tb}|^2 \, |C_7^{eff}(\mu)|^2, \tag{6}$$

in the mass scale μ of the decaying particle, namely the b-quark. The calculation of $C_7(\mu)$ has been done by many authors [38], and we just quote the results. The corrections of $O(\alpha_s)$, due to gluon bremsstrahlung for $m_t > M_W$ have been taken into account. The running of the coupling constant, evaluated at the m_b scale, has been matched to the value at the W-mass scale. Taking $\lambda = \alpha_s(M_W)/\alpha_s(\mu)$, one has

$$\begin{aligned} C_7(\mu) &= 1.0hC_7(m_t, M_W)_{SM} + 2.66667(g-h)C_8(m_t, M_W)_{SM} \\ &\quad +(-0.4286a - 0.0714b - 0.6494c - 0.038d - 0.0186e \\ &\quad -0.0057f + 2.2996g - 1.08797h)C_2(m_t, M_W)_{SM}, \end{aligned} \tag{7}$$

where one has the SM values of the Wilson coefficients collected from Ref. 39 at the M_W scale as a function of $x = m_t^2/M_W^2$, written as $C_i(m_t, M_W)$, which are functions of mass of t-quark and the mass of W-boson, and are given by

$$\begin{aligned} C_2(m_t, M_W)_{SM} &= -1, \\ C_7(m_t, M_W)_{SM} &= \frac{8x^3 + 5x^2 - 7x}{24(x-1)^3} - \frac{3x^3 - 2x^2}{4(x-1)^4} \ln x, \\ C_2(m_t, M_W)_{SM} &= +\frac{x^3 - 5x^2 - 2x}{8(x-1)^3} + \frac{3x^2}{4(x-1)^4} \ln x, \end{aligned} \tag{8}$$

and $a = \lambda^{0.26086}, b = \lambda^{-0.521739}, c = \lambda^{0.4086189}, d = \lambda^{-0.42298}, e = \lambda^{-0.899393}, f = \lambda^{0.1456}, g = \lambda^{0.608695}, h = \lambda^{0.695654}$.

2.1. Trilinear gauge coupling

The trilinear gauge coupling of the photon to $\mathrm{W}^+\mathrm{W}^-$ may be tested in the b $\rightarrow$ sγ decay in the following way. Anomalous gWW vertices can be investigated by finding the deviations from the SM in tree-level processes, such as $\mathrm{e}^+\mathrm{e}^- \rightarrow \mathrm{W}^+\mathrm{W}^-$ and $\mathrm{p\bar{p}} \rightarrow \mathrm{W}\gamma$, or by their influence on loop-order processes , e.g., the $g-2$ of the muon. In the latter case, cut-offs must be used in order to regulate the the divergent loop integrals and can introduce errors by attributing a physical significance to the cut-off [40]. However, the loop processes, such as b $\to$ sγ decay, avoid this problem due to the GIM cancellation mechanism, and give cut-off-independent bounds on anomalous couplings.

The general CP-conserving Lagrangian for WWW interaction can be written as [41]

$$\mathcal{L}_{WWW} = L_1 + L_2,$$

where

$$L_1 = \mathrm{i} g_{WWW} \left[(W^\dagger_{\mu\nu} W^\mu V^\nu - W^\dagger_\mu V_\nu W^{\mu\nu}) + \kappa_V W^\dagger_\mu W_\nu V^{\mu\nu} \right],$$

$$L_2 = \mathrm{i} g_{WWW} \left[+ \frac{\lambda_V}{M_W^2} W^\dagger_{\lambda\mu} W^\mu_\nu V^{\nu\lambda} - \mathrm{i} g_5^V \epsilon^{\mu\nu\lambda\rho} (W^\dagger_\mu \partial_\lambda W_\nu - W_\nu \partial_\lambda W^\dagger_\mu) V_\rho \right],$$

where symbolically $\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{F}_\nu - \partial_\nu \mathcal{F}_\mu$, $g_{WWW} = g c_w(e)$ for $V_\mu = Z_\mu(A_\mu)$, and the parameters ($\Delta\kappa_V \equiv \kappa - 1$) take on the values $\Delta\kappa, \lambda_V$ and $g_5^V = 0$ in the SM.

We can now write the CP-conserving Lagrangian for WWγ interaction as

$$\mathcal{L}_{anomalous\ trilinear\ gauge\ couplings} = \mathrm{i}\,(W^\dagger_{ab} W^a A^b - W^\dagger_a A_b W^{ab}) + \mathrm{i}\,\phi_\gamma W$$

(9) where symbolically $\mathcal{G}_{\mu\nu} = \partial_\mu \mathcal{G}_\nu - \partial_\nu \mathcal{G}_\mu$, and the two parameters ϕ_γ and ψ_γ are such that $\phi_\gamma = 1$ and $\psi_\gamma = 0$ in the SM. To make the calculation, we adopt the following procedure. The coupling presented in Eq. (9) is put into the diagram in which the photon is emitted from the internal top-quark line and we extract the pure dipole-like terms after performing the loop integrations. All other potential Lorentz structures vanish due to the electromagnetic gauge invariance, and because the photon is on-shell. Now, the operator O_7 of the Hamiltonian, that is the coefficient of the dipole b $\to$ s transition operators, is modified due to the presence of the anomalous trilinear gauge couplings. At the M_W scale, O_7 is the only operator mediating the b $\to$ sγ decay. But mixing occurs between various b $\to$ s transition operators during the evolution of the coefficient of O_7 to the b-quark mass scale. In the M_W scale, only the Wilson coefficient $C_7(m_t, M_W)$ is changed, and one has [42]

$$\begin{aligned} &C_7^{eff}(m_t, M_W) = \\ &C_7(m_t, M_W)_{SM} + (\phi_\gamma - 1) G_1(m_t, M_W) + \psi_\gamma G_2(m_t, M_W) \equiv A\phi + B\psi + C, \end{aligned} \tag{10}$$

where $\phi = \phi_\gamma - 1, \psi = \psi_\gamma$, and

$$G_1(m_t, M_W) = \frac{x^2(x-3)}{4(1-x)^3} \ln x - \frac{x}{2(1-x)^2} \equiv A, \tag{11}$$

$$G_2(m_t, M_W) = -\frac{x^2}{4(1-x)^3} \ln x - \frac{x^2 + x}{2(1-x)^2} \equiv B, \tag{12}$$

Using the lifetime of the B-meson [13] and the limits prescribed by the CLEO data [23], the constraints on ϕ and ψ can be estimated.

2.2. Fourth generation

At this stage, the effects of the fourth generation of quark are introduced. The introduction of the effect of the virtual exchange of the fourth generation up quark t' has been discussed in Ref. 43. Here, we utilize that approach. The Wilson coefficients of the dipole operators at the W-boson mass scale, in the limit of vanishing up and charm masses, can be written simply by replacing the term $V_{ts}^* V_{tb} C_i(m_t, M_W)$ by

$$V_{ts}^* V_{tb} C_i(m_t, M_W) + V_{t's}^* V_{t'b} C_i(m_{t'}, M_W),$$

where V_{pq} represents the 4×4 CKM matrix elements, and $i = 7, 8$.

So, the fourth generation Wilson coefficient at the W-mass scale can be written as

$$C_{i,\ fourth\ generation}(m_t, M_W) = \\ C_i(m_t, M_W) + \frac{V_{t's}^* V_{t'b}}{V_{ts}^* V_{tb}} C_i(m_{t'}, M_W) \equiv C_i(m_t, M_W) + V C_i(m_{t'}, M_W), \quad \text{for } i = 7, 8$$

Equation (6) now yields $\Gamma(\mathrm{b} \to \mathrm{s}\gamma)$ by replacing $C_7^{eff}(\mu)$ by $C_{7,\ fourth\ generation}^{eff}(\mu)$ which is obtained from Eq. (8) with $C_7(m_t, M_W)_{SM}$ and $C_8(m_t, M_W)_{SM}$ replaced by $C_{7,\ fourth\ generation}(m_t, M_W)$ and $C_{8,\ fourth\ generation}(m_t, M_W)$, respectively. Rewriting $C_7^{eff}(\mu)$ given in (7) as

$$C_7^{eff}(\mu) \equiv A_1 C_7(m_t, M_W)_{SM} + B_1 C_8(m_t, M_W)_{SM} + C_1$$

where $A_1,\ B_1$ and C_1 are functions of λ, one has

$$C_{7,\ fourth\ generation}^{eff} = P + Q\phi + R\psi, \tag{13}$$

where

$$P = A_1[C_7(m_t, M_W)_{SM} + V C_7(m_{t'}, M_W)_{SM}] + \\ B_1[C_8(m_t, M_W)_{SM} + V C_8(m_{t'}, M_W)_{SM}] + C_1, \tag{14}$$

$$Q = A_1[G_1(m_t, M_W) + V G_1(m_{t'}, M_W)], \tag{15}$$

$$R = A_1[G_2(m_t, M_W) + V G_2(m_{t'}, M_W)]. \tag{16}$$

The constraints on ϕ and ψ can now be estimated using results of Refs. 13 and 15.

3. Results and conclusion

Nowadays, it is customary to calculate strong-coupling constants with $\alpha_s(M_Z)$ as an initial condition. A straightforward calculation gives the solution [39] of the form

$$\alpha_s(\mu) = \frac{\alpha_s(M_Z)}{\alpha(\mu)}\left(1 - \frac{\beta_1}{\beta_0}\frac{\alpha_s(M_Z)}{4\pi}\frac{\ln\alpha(\mu)}{\alpha(\mu)}\right) \tag{17}$$

$$\text{where} \quad \alpha(\mu) = 1 - \beta_0\frac{\alpha_s(M_Z)}{2\pi}\ln\left(\frac{M_Z}{\mu}\right). \tag{18}$$

Thus, $\lambda = \alpha_s(M_W)/\alpha_s(\mu)$ can now be calculated.

For the calculation, in the present case, $\beta_0 = (11N - 2n_f)/3$, and $\beta_1 = (34/3)N^2 - (10/3)Nn_f - 2c_f n_f$, with N the number of colours, n_f the number of active flavours and $c_f = (N^2-1)/(2N)$. Here, $N = 3$ and $n_f = 5$. The following values were taken [44]: $\alpha^{-1} = 137.036, G_F = 1.166372\times10^{-5}$ GeV^2 and $M_Z = 91.187$ GeV, and $M_W = 80.33$ GeV has been used.

The values of $\alpha_s(M_Z^2)$ so far available are: (i) 0.118 ± 0.007 [44], (ii) 0.118 ± 0.006 [45], (iii) 0.1212 ± 0.0044 from the $O(\alpha_s^2)$ analysis 0.1131 ± 0.0028 from the $O(\alpha_s^3)$ analysis [46], and recently the Giele-determined value [47] by the "evolution rate" of the parton-density functions rather than by the "event rate" as

$$\begin{aligned} \alpha_s^{LO}(M_Z) &= 0.110 \pm 0.001(\mathrm{stat}) \pm 0.004(\mathrm{theory}) \\ \alpha_s^{NLO}(M_Z) &= 0.114 \pm 0.001(\mathrm{stat}) \pm 0.004(\mathrm{theory}) \end{aligned} \tag{19}$$

The average of all these prescriptions, vis. 0.1157, has been taken in our calculation.

The values of $\alpha_s(\mu)$ at the M_Z scale for different values of m_b are calculated with the help of Eqs. (17) and (18). The calculation of λ is trivial.

We use Eq. (6) to calculate $\Gamma(\mathrm{b} \to \mathrm{s}\gamma)$, and using the mean life of B [13], one has the total decay width $\Gamma_b = 4.274105 \times 10^{-13}$ GeV. The branching ratio is calculated for different values of m_t and m_b. The results are given in Table 1.

TABLE 1. Branching ratio $Br(\mathrm{b} \to \mathrm{s}\gamma)$, and upper and lower limits of ϕ and ψ for a set of assumed values of m_t and m_b.

m_t (GeV)	m_b (GeV)	$Br(\mathrm{b} \to \mathrm{s}\gamma)$ In units of 10^{-4}	ϕ		ψ	
			Lower limit	Upper limit	Lower limit	Upper limit
180	5.0	1.814	0.6779	0.6934	1.0275	1.0511
	4.6	1.229	0.6907	0.7099	1.0470	1.0760
175	5.0	1.787	0.6891	0.7051	0.9945	1.0175
	4.6	1.211	0.7024	0.7220	1.0136	1.0419
170	5.0	1.760	0.7011	0.7174	0.9592	0.9815
	4.6	1.194	0.7147	0.7349	0.9778	1.0053
160	5.0	1.703	0.7273	0.7445	0.8803	0.9011
	4.6	1.156	0.7419	0.7631	0.8979	0.9235

But this does not include the SM values, and so we also calculate the upper and lower limits of ϕ for the nonvanishing value of ψ, the SM value, and the same for ψ for the vanishing value of ϕ. This is given in Table 2.

TABLE 2. The SM values of $\phi(\psi = 0)$ and $\psi(\phi = 0)$ for a set of assumed values of m_t and m_b.

m_t (GeV)	m_b (GeV)	$\phi(\psi = 0)$		$\psi(\phi = 0)$	
		Lower limit	Upper limit	Lower limit	Upper limit
180	5.0	-0.32	1.64	-0.48	0.25
	4.6	-0.539	1.88	-0.81	0.09
175	5.0	-0.33	1.68	-0.48	0.24
	4.6	-0.55	1.92	-0.804	0.09
170	5.0	-0.34	1.71	-0.47	0.23
	4.6	-0.57	1.96	-0.78	0.08
160	5.0	-0.37	1.75	-0.45	0.20
	4.6	-0.61	2.06	-0.75	0.07

With $m_t = 180$ GeV and $m_b = 4.6$ GeV, keeping the lower limit of ϕ fixed, the allowable region for ϕ is found to be between 0.0002 and 0.91.

To observe the effects of the fourth generation, we calculate the branching ratio for the $m_t = 180$ GeV and $m_b = 4.6$ GeV with different values of the fourth-generation quark masses, and also the limits of ϕ and ψ as constrained by the CLEO data. The results are given in Table 3.

TABLE 3. Branching ratio $Br(\mathrm{b} \to \mathrm{s}\gamma)$ for $m_t = 180$ GeV and $m_b = 4.6$ GeV, for different values of the fourth-generation quark masses, and the limits on ϕ and ψ, constrained by the CLEO data.

m_t (GeV)	$m_{t'}$ (GeV)	$Br(\mathrm{b} \to \mathrm{s}\gamma)$ In units of 10^{-4}	ϕ		ψ	
			Lower limit	Upper limit	Lower limit	Upper limit
45	110	1.78	0.884	0.905	0.643	0.657
50	110	1.82	0.879	0.899	0.626	0.641
	120	1.81	0.875	0.895	0.821	0.839
60	110	1.88	0.869	0.889	0.599	0.612
	120	1.87	0.866	0.885	0.792	0.809
	130	1.86	0.862	0.881	0.939	0.960
85	110	2.00	0.849	0.868	0.551	0.564
	120	2.05	0.838	0.856	0.722	0.738
	130	1.97	0.845	0.864	0.895	0.915
	140	2.11	0.820	0.838	0.957	0.978
90	110	2.01	0.850	0.869	0.551	0.563
	120	1.99	0.844	0.863	0.740	0.756
	130	1.98	0.841	0.859	0.888	0.907
	140	1.97	0.837	0.855	1.002	1.024
	150	1.96	0.835	0.853	1.085	1.108

For the masses of $m_{t'}$ and $m_{b'}$ above the top-quark mass, no appreciable variation has been observed. The values obtained for the branching ratio with the effect of the fourth generation

for $\quad m_{t'} = 500$ GeV, $m_{b'} = 400$ GeV, $\quad Br(\mathrm{b} \to \mathrm{s}\gamma) = 1.229023 \times 10^{-4}$,

and

for $\quad m_{t'} = 540$ GeV, $m_{b'} = 500$ GeV, $\quad Br(\mathrm{b} \to \mathrm{s}\gamma) = 1.229020 \times 10^{-4}$,

and the corresponding of P, Q and R do not undergo sufficient change. The limits of ϕ are 1.037 to 1.066, and of ψ 1.572 to 1.616.

From the above results, we can safely conclude that the allowable region for ϕ is from -0.61 to 2.06, and that for ψ from -0.81 to 0.25.

What is interesting in this formulation is that we get a different region for the allowable range than predicted in Ref. 43, but at the same time it conforms to the CLEO data.

This formulation is done keeping in mind that a fourth generation is consistent with the LEP/SLC data as long as the fourth neutrino is heavy, i.e., $m_{\nu_4} \geq M_Z/2$, and such a heavy fourth neutrino could mediate a see-saw-type mechanism, thus generating a small mass for ν_e, ν_μ and ν_τ. The possibility of a fourth family of fermions may be taken as a popular extension.

Appendix

The following method has been used for the calculation of the fourth generation CKM-mixing matrices from quark masses. It contains six angles $\theta_i, i =$

$1, \ldots, 6$, $0 \leq \theta_i \leq \pi/2$, and four CP-nonconserving phases δ_i, $i = 1, 2, 3$, $\pi \leq \delta_i \leq \pi$, such that the fourth-generation matrix reduces to three-generation CKM matrix [1] when $\theta_4 = \theta_5 = \theta_6 = 0$ and $\delta_2 = \delta_3 = 0$. This is given below [48]:

$$V = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_4 & -s_4 \\ 0 & 0 & s_4 & c_4 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_2 & -s_2 & 0 \\ 0 & s_2 & c_2 & 0 \\ 0 & 0 & 0 & \mathrm{e}^{\mathrm{i}\delta_2} \end{pmatrix} \begin{pmatrix} c_1 & s_1 & 0 & 0 \\ -s_1 & c_1 & 0 & 0 \\ 0 & 0 & \mathrm{e}^{\mathrm{i}\delta_1} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\times \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_6 & s_6 \\ 0 & 0 & -s_6 & c_6 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_3 & s_3 & 0 \\ 0 & -s_3 & c_3 & 0 \\ 0 & 0 & 0 & \mathrm{e}^{\mathrm{i}\delta_3} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_5 & s_5 \\ 0 & 0 & -s_5 & c_5 \end{pmatrix}$$

where $s_1 = \sin\theta_i$ and $c_1 = \cos\theta_i$. So we have:
$V_{ud} = c_1, \quad V_{us} = s_1 c_3, \quad V_{ub} = s_1 s_3 c_5, \quad V_{ub'} = s_1 s_3 s_5, \quad V_{cd} = s_1 c_2,$
$V_{cs} = c_1 c_2 c_3 + s_2 s_3 c_6 \mathrm{e}^{\mathrm{i}\delta_1}, \quad V_{cb} = c_1 c_2 s_3 c_5 - s_2 c_3 c_5 c_6 \mathrm{e}^{\mathrm{i}\delta_1} + s_2 s_5 s_6 \mathrm{e}^{\mathrm{i}(\delta_1+\delta_3)},$
$V_{cb'} = c_1 c_2 s_3 s_5 - s_2 c_3 c_5 c_6 \mathrm{e}^{\mathrm{i}\delta_1} - s_2 c_5 s_6 \mathrm{e}^{\mathrm{i}(\delta_1+\delta_3)},$
$V_{td} = -s_1 s_2 c_4, \quad V_{ts} = c_1 s_2 c_3 c_4 - c_2 s_3 c_4 c_6 \mathrm{e}^{\mathrm{i}\delta_1} - s_3 s_4 s_6 \mathrm{e}^{\mathrm{i}\delta_2},$
$V_{tb} = c_1 s_2 s_3 c_4 c_5 + c_2 c_3 c_4 c_5 c_6 \mathrm{e}^{\mathrm{i}\delta_1} + c_2 c_4 s_5 s_6 \mathrm{e}^{\mathrm{i}(\delta_1+\delta_3)} + c_3 s_4 c_5 s_6 \mathrm{e}^{\mathrm{i}\delta_2} + s_4 s_5 c_6 \mathrm{e}^{\mathrm{i}(\delta_2+\delta_3)},$
$V_{tb'} = c_1 s_2 s_3 c_4 s c_5 + c_2 c_3 c_4 s_5 c_6 \mathrm{e}^{\mathrm{i}\delta_1} + c_2 c_4 c_5 s_6 \mathrm{e}^{\mathrm{i}(\delta_1+\delta_3)} + c_3 s_4 s_5 s_6 \mathrm{e}^{\mathrm{i}\delta_2} + s_4 s_5 c_6 \mathrm{e}^{\mathrm{i}(\delta_2+\delta_3)},$
$V_{t'd} = -s_1 s_2 s_4, \quad V_{t's} = c_1 s_2 c_3 s_4 - c_2 s_3 c_4 c_6 \mathrm{e}^{\mathrm{i}\delta_1} - s_3 c_4 s_6 \mathrm{e}^{\mathrm{i}\delta_2},$
$V_{t'b} = c_1 s_2 s_3 s_4 c_5 + c_2 c_3 s_4 c_5 c_6 \mathrm{e}^{\mathrm{i}\delta_1} - c_2 s_4 s_5 s_6 \mathrm{e}^{\mathrm{i}(\delta_1+\delta_3)} + c_3 c_4 c_5 c_6 \mathrm{e}^{\mathrm{i}\delta_2} + c_4 s_5 c_6 \mathrm{e}^{\mathrm{i}(\delta_2+\delta_3)},$
$V_{t'b'} = c_1 s_2 s_3 s_4 s_5 + c_2 c_3 s_4 s_5 c_6 \mathrm{e}^{\mathrm{i}\delta_1} - c_2 s_4 s_5 s_6 \mathrm{e}^{\mathrm{i}(\delta_1+\delta_3)} - c_3 c_4 s_5 s_6 \mathrm{e}^{\mathrm{i}\delta_2} + c_4 c_5 c_6 \mathrm{e}^{\mathrm{i}(\delta_2+\delta_3)}.$

Now, to have an estimate of the CKM matrix elements, the following matrices are taken:

$$M^{\mathrm{up}} = \begin{pmatrix} 0 & \alpha_1 & 0 & 0 \\ \alpha_1 & 0 & \alpha_1 & 0 \\ 0 & \alpha_2 & 0 & \alpha_3 \\ 0 & \alpha_3 & \alpha_4 & 0 \end{pmatrix}$$

and

$$M^{\mathrm{down}} = \begin{pmatrix} 0 & \beta_1 & 0 & 0 \\ \beta_1 & 0 & \beta_2 & 0 \\ 0 & \beta_2 & 0 & \beta_3 \\ 0 & 0 & \beta_3 & \beta_4 \end{pmatrix}.$$

Also, $\gamma_1^{up} = -m_u, \gamma_2^{up} = m_c, \gamma_3^{up} = -m_t$ and $\gamma_4^{up} = m_{t'}$. Then, one gets:

$$\begin{aligned} \alpha_4 &= \sum_{i=1}^{4} \gamma_i, \\ (\alpha_1^2 + \alpha_2^2)\alpha_4 &= -\sum_{i<j<k} \gamma_i \gamma_j \gamma_k, \end{aligned}$$

$$\alpha_1^2 + \alpha_2^2 + \alpha_3^2 = -\sum_{i<j} \gamma_i \gamma_j ,$$

$$\alpha_1^2 \cdot \alpha_2^2 = \gamma_1 \gamma_2 \gamma_3 \gamma_4 ,$$

$V(\text{the CKM matrix}) = U^u U^{d\,\dagger}$.

The unnormalized i^{th} row of U^u, corresponding to up-type quarks, is given by

$$1,\ \gamma_i/\alpha_1,\ (\gamma_i^2 - \alpha_1^2)/(\alpha_1\alpha_2),\ \gamma_i(\gamma_i^2 - \alpha_1^2 - \alpha_2^2)/(\alpha_1\alpha_2\alpha_3).$$

The calculation of U^d for the down-type quarks is similar. Thus, for a set of masses in respect of u, c, t, t' and d, s, b, b', one can calculate all the sixteen matrix elements $V_{ij}, i =$u, c, t, t' and $j =$d, s, b, b'.

The set of masses (current quark mass) taken for the computations are:

$$\text{up}: \quad m_u = 0.005\,\text{GeV}, \quad m_c = 1.3\,\text{GeV}, \quad m_t = 180\,\text{GeV},$$

and the values of $m_{t'}$ were taken as stated earlier,

$$\text{down}: \quad m_d = 0.01\,\text{GeV}, \quad m_s = 0.2\,\text{GeV}, \quad m_b = 4.6\,\text{GeV},$$

and the values of $m_{b'}$ were taken as stated earlier.

It may be noted that the CP violation phases δ_i have been assumed to be approximately equal 0, or negligibly small, and the elements of $U^{u(d)}$ are functions of masses. However, very small mass ratios are neglected so that the mass dependence of mixing angles becomes simpler, and the matrix elements are calculated up to the nine decimal places, otherwise the mixing angles can not be estimated reliably. This is also done to avoid the absurd results like $\cos\theta > 1$.

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OGRANIČENJE TRILINEARNOGA BAŽDARNOG VEZANJA RASPADOM $b \to s\gamma$ UZEVŠI U OBZIR MOGUĆNOST ČETVRTE GENERACIJE KVARKOVA

Virtualni učinci trilinearnoga baždarnog vezanja fotona na W^+W^- na raspad $b \to s\gamma$ se provjeravaju uz pretpostavku moguće četvrte generacije kvarkova, s popravkama do vodećih QCD logaritama, primjenom razvoja CKM matrice četvrte generacije i fazom kršenja CP-invarijantnosti jednakom nula. Uzeta su u obzir ograničenja izmjerenom vrijednošću parametra ρ.