The structure of the nucleon can be described by the electromagnetic form factors $G_E$ and $G_M$. These form factors can be determined by elastic electron-proton scattering with the well-known Rosenbluth technique. Alternatively, the ratio of the electric and the magnetic form factors $G_E/G_M$ can be determined by the so-called polarization-transfer technique. The measurements reveal a large discrepancy between the two methods at large momentum transfers. There is evidence that two-photon exchange contributions to the elastic scattering have been underestimated. While the two-photon exchange amplitude is difficult to compute from theory due to the excited intermediate hadronic states, its imaginary part can be accessed experimentally by measuring the asymmetry $A_\perp$ in the cross section of elastic $ep$ scattering where the electrons are transversely polarized parallel or antiparallel to the normal of the scattering plane, respectively. The PVA4 experiment at MAMI is currently performing such measurements covering a large range of momentum transfers.

1. Introduction

Elastic electron-nucleon scattering is a widely used tool to study the structure of the nucleon. The measurements are usually interpreted in the Born approximation where one virtual photon is exchanged (Fig. 1).

The structure of the nucleon can be parameterized by the Pauli and Dirac form factors $F_1$ and $F_2$ $N(p)$

$$\langle J^\mu \rangle = -ie \bar{N}(p') \left[ \gamma^\mu F_1(q^2) + i \frac{q^\mu q_\nu}{2M} F_2(q^2) \right] N(p), \quad (1)$$

where $\langle J^\mu \rangle$ represents the hadronic current. Two different approaches exist to mea-
sure these form factors. One is the well-known Rosenbluth separation method, the other the so-called polarization-transfer technique where the ratio of electric and magnetic form factor $G_E/G_M$ is determined. The two methods reveal a significant discrepancy at higher momentum transfers [1]. A possible explanation for this discrepancy is that the impact of the two-photon exchange might have been underestimated in the past [2]. This article will introduce the formalism of the two-photon exchange amplitude and present measurements of its real part. The main focus will be on the measurements of the imaginary part which are done by the A4 collaboration at the MAMI accelerator facility.

Fig. 1 (left). Elastic electron-nucleon scattering in the Born approximation.
Fig. 2. Two-photon contributions to the scattering process (diagrams at right).

2. Nucleon form factors and the two-photon exchange amplitude (TPE)

In order to include the exchange of two virtual photons (Fig. 2), the scattering amplitude is described by six complex functions $\hat{G}_M, \hat{F}_2, \hat{F}_3, \hat{F}_4, \hat{F}_5$ and $\hat{F}_6$ [3]. In the Born approximation these functions reduce to the usual form factors:

\[
\begin{align*}
\hat{G}_M^{\text{Born}}(s, Q^2) &= G_M(Q^2), \\
\hat{F}_2^{\text{Born}}(s, Q^2) &= F_2(Q^2), \\
\hat{F}_3^{\text{Born}} &= 0.
\end{align*}
\]

These functions cannot be calculated straightforwardly since hadronic contributions are involved. Not only the proton ground state, but also intermediate excited states contribute. Calculations for the imaginary part were made in Ref. [3]. A measurement of these quantities gives hence insight into the hadron structure. Both the real part and the imaginary part can be accessed by measurements.

3. Measurements of the real part of the TPE

For a description of elastic electron-proton scattering, which includes the exchange of more than one virtual photon, it is suitable to use the following quantities:

\[
\begin{align*}
\hat{G}_E &= G_E(Q^2) + \delta \hat{G}_E, \\
\hat{G}_M &= G_M(Q^2) + \delta \hat{G}_M, \\
\hat{F}_3.
\end{align*}
\]
\( \delta \hat{G}_E, \delta \hat{G}_M \) and \( \hat{F}_3 \) expand the nucleon form factors and originate from processes involving the exchange of at least two photons. The reduced cross section can then be written [4] as

\[
\sigma_R = G_M^2 + \frac{e}{\tau} G_E^2 + 2G_M \Re \left( \delta \hat{G}_M + \frac{\nu}{M^2} \hat{F}_3 \right) + 2G_E \Re \left( \delta \hat{G}_E + \frac{\nu}{M^2} \hat{F}_3 \right) + O(e^4),
\]

(2)

One can see from Eq. 2 that only the real part of the TPE enters here. This real part of the TPE can be measured by comparing the cross section for elastic electron-proton and elastic positron-proton scattering. There was an early experiment at SLAC in 1968 [6] which could not resolve two-photon effects within the precision of the measurement. New measurements are underway or in development at various accelerator facilities: at VEPP-3 [7], at Jefferson Lab [8] and at DESY [9].

### 4. Measurement of the imaginary part of the TPE

If the electrons are transversely polarized, an asymmetry in the cross section of elastic \( ep \)-scattering arises from the interference of the one-photon and the two-photon exchange amplitude. The asymmetry is defined as

\[
A_{\text{Spin}} = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} = A_{\perp} \frac{\vec{S}_n \cdot \vec{P}}{|\vec{S}_n| |\vec{P}|} = A_{\perp} \sin \varphi_e,
\]

(3)

where \( \sigma_+ \) and \( \sigma_- \) denote the cross sections for right and left transversely polarized electrons. See Fig. 3 for the definitions of the other quantities. The asymmetry depends on the azimuthal scattering angle and shows a modulation with \( \sin(\varphi_e) \).

![Fig. 3. Scattering of transversely polarized electrons. The incoming momentum is denoted by \( \vec{k} \), the outgoing by \( \vec{k}' \). The scattering plane is shown in blue, the vector \( \vec{S}_n \) is normal vector of the scattering plane. The spin of the electron before the scattering is shown by \( \vec{P} \).](image-url)
The asymmetry is sensitive on the imaginary part of the TPE [3]:

\[
A_\perp = \frac{2m_e}{Q} \sqrt{2\tau(1-\epsilon)} \sqrt{1 + \frac{1}{\tau} \left(G_M^2 + \frac{\epsilon G_E^2}{\tau}\right)^{-1}} \\
\times \left\{ -\tau G_M \Im \left( \hat{F}_3 + \frac{1}{1 + \frac{\nu}{M^2}} \hat{F}_5 \right) - G_E \Im \left( \hat{F}_4 + \frac{1}{1 + \frac{\nu}{M^2}} \hat{F}_5 \right) \right\} \\
+ O(\epsilon^4).
\]

The asymmetries depend on the scattering angle and the beam energy and are in the order of $10^{-5}$. These asymmetries can be measured by PV experiments. So far four groups have published results:

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>SAMPLE</td>
<td>0.192</td>
<td>0.10</td>
<td>-16.4 ± 5.9</td>
</tr>
<tr>
<td>A4</td>
<td>0.570</td>
<td>0.11</td>
<td>-8.59 ± 0.89</td>
</tr>
<tr>
<td>A4</td>
<td>0.855</td>
<td>0.23</td>
<td>-8.52 ± 2.31</td>
</tr>
<tr>
<td>HAPPEX</td>
<td>3.0</td>
<td>0.11</td>
<td>-6.7 ± 1.5</td>
</tr>
<tr>
<td>G0</td>
<td>3.0</td>
<td>0.15</td>
<td>-4.06 ± 1.62</td>
</tr>
<tr>
<td>G0</td>
<td>3.0</td>
<td>0.25</td>
<td>-4.82 ± 2.85</td>
</tr>
</tbody>
</table>

The PV experiment at MAMI will be described in more detail. The A4 experiment [10] takes place at the electron accelerator facility MAMI at Mainz [11]. A superlattice photocathode delivers a polarized electron beam with a current of 20 µA and an average polarization of about 80%. The electron beam is accelerated up to $E = 1.5$ GeV. The spin of electrons is controlled via a Wien filter, both longitudinal and transverse beam polarizations are possible for all beam energies. The spin direction of the electrons is changed every 20 ms following a randomly selected pattern of either “+−−+” or “−+−+”. To suppress false asymmetries which may arise due to different beam position, angle, intensity or energy for the two polarization states, several feedback stabilization systems are installed along the way of the electrons from the accelerator to the target. The liquid hydrogen target [13] is about 10 cm long for the forward angle measurements and about 23 cm long for the backward angle measurement giving a luminosity of $L \approx 0.5 \cdot 10^{38}$ cm$^{-2}$s$^{-1}$ or $L \approx 1.2 \cdot 10^{38}$ cm$^{-2}$s$^{-1}$, respectively.

The scattered electrons are detected in a homogenous electromagnetic calorimeter that consists of 1022 lead fluoride crystals (see Fig. 4). Lead fluoride is a pure Cherenkov radiator and hence intrinsically fast. The whole calorimeter can cope with event rates of 100 MHz. The energy resolution is about $\Delta E/E = 3.9%/\sqrt{E[\text{GeV}]}$. The covered angles are $2\pi$ in the azimuthal range and...
Additional scintillators

\( \text{PbF}_2 \) crystals

\( \text{H}_2 \) target

Fig. 4. Drawing of the A4 lead fluoride calorimeter and the scattering chamber [12].

The calorimeter is mounted on a rotatable platform so that one can easily change between forward and backward angle configuration. The backward angle configuration is shown here. The lead fluoride crystals cover the \( 2\pi \) azimuthal range. Between the crystals and the scattering chambers are plastic scintillators installed which allow a separation of neutral and charged particles.

\( 30^\circ \leq \theta \leq 40^\circ \) (forward configuration) or \( 140^\circ \leq \theta \leq 150^\circ \) (backward configuration) in the polar range, resulting in a covered solid angle of \( \Delta \Omega = 0.62 \text{ sr} \). In the backward angle configuration, there are 72 plastic scintillators installed in front of the \( \text{PbF}_2 \) crystals. They enable a separation of neutral from charged particles, i.e. in our case photons from \( \pi^0 \)-decay from scattered electrons which cannot be distinguished by the \( \text{PbF}_2 \) calorimeter itself because the photons produce in the crystals electromagnetic showers very similar to those coming from electrons.

A typical energy spectrum is shown in Fig. 5. The elastic peak is clearly visible. By applying cuts around the elastic peak, one can determine the number of elastically scattered electrons for the two polarization states and hence extract the asymmetry. Each of the 146 frames of the calorimeter corresponds to a specific azimuthal scattering angle \( \varphi_e \). For each frame \( i \), the asymmetry \( A_i \) is determined. Corrections for false asymmetries due to fluctuations in the beam properties like intensity, position or energy and for the beam polarization have to be applied. To extract the physical asymmetry out of the measured asymmetry, we perform a multiple linear regression using the ansatz

\[
A_{i,\text{meas}} = P \cdot A_{i,\text{phys}} + \sum_{j=1}^{6} a_{ij} \cdot X_j ,
\]

with \( P \) the electron beam polarization, \( X_j \) are the polarization-depending beam differences and \( a_{ij} \) the correlation coefficients which are determined by the regression analysis. Figure 6 shows an example of extracted asymmetries for backward angles.
beam energy was 1.5 GeV.

Fig. 5. Energy spectrum measured by the A4 lead fluoride calorimeter. The incident beam energy was 1.5 GeV and a hydrogen target was used. The x axis shows ADC channels. One can see the peak of elastically scattered electrons around ADC threshold and the position of the Δ resonance are also indicated.

![Energy Spectrum](image)

a hydrogen target and a beam energy of 315 MeV. Due to other conventions concerning the electron scattering angle φₑ by the experiment, there is a phase shift of π/2 compared to the theorist definition in Eq. (3). By determination of the amplitude of the cosine, one can extract the quantity A⊥.

In the A4 experiment, measurements were made with different beam energies, at forward and backward angles. At backward angles, also deuterium was used as target. Each single measurement corresponds to a data taking time of 2-7 days.

Fig. 6. Measured asymmetries A⊥ at backward angles, hydrogen target and beam energy E = 315 MeV, as a function of the frame number of the calorimeter. The frames cover the whole azimuthal angle, i.e. frame 146 corresponds to φₑ = 2π.

![Asymmetry Graph](image)
The beam energies, targets and kinematics that are covered by the A4 program are summarized in Table 1.

The analysis of the data is ongoing. Figure 7 shows calculations of the asymmetry from Ref. [3] together with two already published results and two new preliminary results at forward angles.

**TABLE 1. Two-photon program of the A4 collaboration at MAMI. Most of the measurements are already done right now.**

<table>
<thead>
<tr>
<th>Beam energy</th>
<th>Kinematics</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>315 MeV</td>
<td>forward</td>
<td>H_2</td>
</tr>
<tr>
<td>315 MeV</td>
<td>backward</td>
<td>H_2</td>
</tr>
<tr>
<td>315 MeV</td>
<td>backward</td>
<td>D_2</td>
</tr>
<tr>
<td>420 MeV</td>
<td>forward</td>
<td>H_2</td>
</tr>
<tr>
<td>420 MeV</td>
<td>backward</td>
<td>H_2</td>
</tr>
<tr>
<td>420 MeV</td>
<td>backward</td>
<td>D_2</td>
</tr>
<tr>
<td>510 MeV</td>
<td>forward</td>
<td>H_2</td>
</tr>
<tr>
<td>570 MeV</td>
<td>forward</td>
<td>H_2</td>
</tr>
<tr>
<td>855 MeV</td>
<td>forward</td>
<td>H_2</td>
</tr>
<tr>
<td>1508 MeV</td>
<td>forward</td>
<td>H_2</td>
</tr>
</tbody>
</table>

*Fig. 7. Transverse asymmetry $A_\perp$ as a function of the beam energy for the A4 experiment at forward angles. The curves show the calculation from [3]: Nucleon ground state only (green), excited intermediate states (blue) and the sum of the two (red). The data points show measurements at MAMI. The points in black are taken from [5], the red points are preliminary.*
5. Conclusions and outlook

Two-photon physics is an active field in research today. For the real part of the TPE, measurements at VEPP-3 are underway right now. The OLYMPUS experiment at DESY will provide precise data in a few years. Concerning the imaginary part, results are already available by various PV experiments. Additional results by the G0 collaboration are expected soon. The A4 collaboration has performed a rich measurement program. The preliminary data show that the excited intermediate hadronic states play an important role.

Altogether, the experimental efforts will improve substantially our understanding of two-photon effects in elastic electron-nucleon scattering. One can see that the ground state contribution to the asymmetry \( A_\perp \) is small for beam energies above 200 MeV. This behaviour is confirmed by the asymmetries that were measured by the A4 experiment. However, the size of the measured asymmetries in the energy region between 300 MeV and 600 MeV is distinctly smaller than the result of the calculation.

References