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WAVELET ANALYSIS OF MULTIPARTICLE CORRELATIONS

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Applicability of continuous wavelet analysis to pseudorapidity and azimuthal distributions of secondary particles produced in Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV at Relativistic Heavy Ion Collider is studied. After corrections for boundary effects, the wavelet spectra are presented on different scales. We present a discussion whether the multi-particle collective flows produced by different physical mechanisms can be detected through extremes in the wavelet spectra or corresponding scalograms, and what additional information concerning correlation structure of events can be extracted by this analysis.

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1. *Introduction*

The primary goal of ultrarelativistic heavy-ion collisions is to sqeeze and heat up nuclear matter above the critical values predicted by QCD in order to create and study exotic state of nuclear matter called quark-gluon plasma (QGP). Its properties are deduced through a variety of partial signals which are subsequently combined to produce QGP global picture [1]. Information about the partonic collective flows at the early stages of QGP expansion can be retained in the angular spectra of emitted particles and extracted by analysis of multi-particle correlations.

The experimental data sample discussed here was obtained with the STAR detector [2] at the Relativistic Heavy Ion Collider (RHIC) at the Brookhaven National Laboratory. The main detector for the analysis is the Time Projection Chamber (TPC) [3] which is located inside a 0.5 T solenoidal magnet and measures tracks and momenta of charged particles in the pseudorapidity region $|\eta| < 1.3$ with full azimuthal coverage.

As for the selection criteria, the events are selected if their primary vertices lie within 30 cm distance from the TPC center along the z axis (the beam line) and within 1 cm distance along the y axis. The tracks are chosen if their distance of closest approach to the primary vertex is less than 2 cm, and at least 15 measured space points were used for their reconstruction. Furthermore, the ratio of the number of measured space points to the maximum number of space points for each track has to be larger than 0.52 and χ^2 of track fit in the xy plane has to lie between 0.1 and 1.3.

Small testing sample consists of approximately 1500 minimum-bias Au+Au events taken at the collision energy $\sqrt{s_{NN}} = 200$ GeV in the 2004 run.

2. *Wavelet spectra*

Various techniques are employed to study multi-particle correlations. We present the preliminary results obtained by means of the wavelet analysis employing the second derivative of the Gaussian function g_2 as a mother wavelet function. The method description and its application to heavy-ion collisions data are presented at length for instance in Refs. [4], [5] and [6] and in the publications reffered to therein. Similar approach to study multi-particle correlations in STAR data, though based on the discrete wavelet transform, was first time applied in Ref. [7].

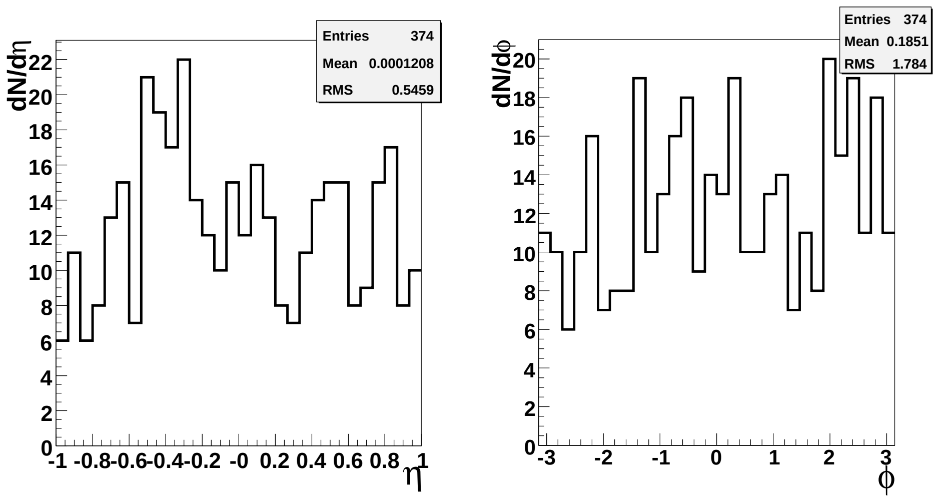
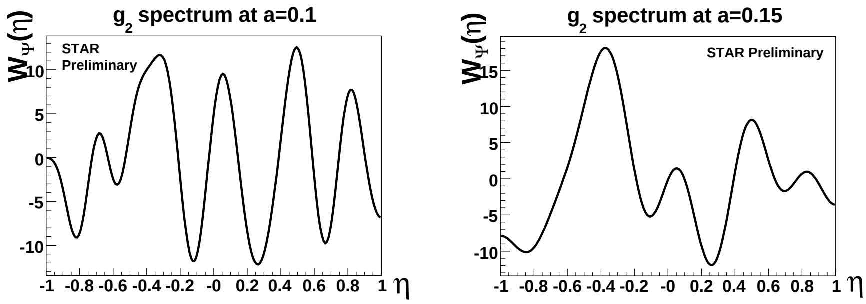


Fig. 1. Pseudorapidity and azimuthal angle distribution of a representative Au+Au 200 GeV *event.*

Figure 1 shows the distributions of pseudorapidities η and azimuthal angles ϕ of particles produced in the representative experimental event with the multiplicity of charged particles close to 370 in the eta range $|\eta| < 1$. The g_2 wavelet η and

ϕ spectra of both distributions seen on the three different scales a are shown in Figs. 2 and 3. Wavelet coefficients on the vertical axes are denoted as W_Ψ.



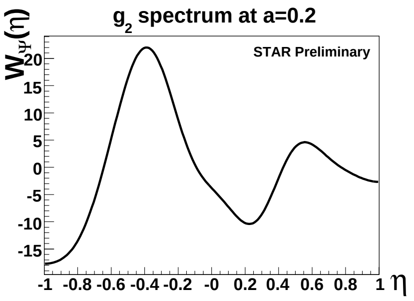
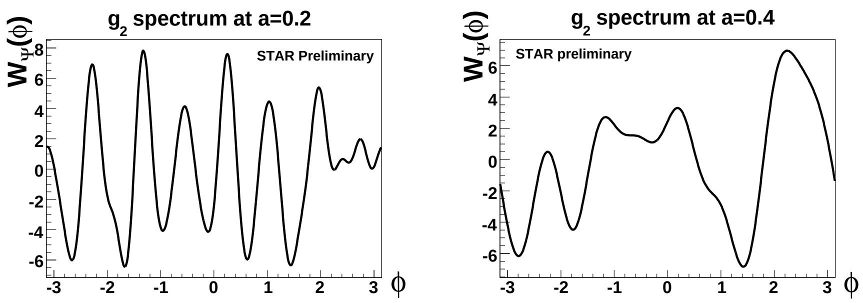


Fig. 2 (above and left). Wavelet pseudorapidity spectrum of the studied event seen on three different scales a.



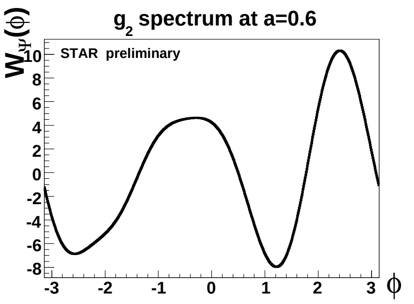


Fig. 3 (above and left). Wavelet azimuthal angle spectrum of the studied event seen at the three different scales a.

Local maxima visible in the wavelet spectra imply prevailing emission angles of groups of secondary particles [4, 8]. Therefore, the presented plots demonstrate the evolution of clusterization of particles. On the finest scales, only individual particles or small groups of particles are observed. The relatively large numbers of maxima reveal evident irregularity of the η and ϕ distributions on the smallest scales. When going to larger scales, the particle groups gradually merge, eventually producing both in η and ϕ cases two large particle flows which move in opposite directions. The relationship among the peaks observed in the η and ϕ wavelet spectra can be studied only by two-dimensional wavelet method or other type of two-dimensional correlation analysis (or their combination).

When combining many plots, like those in the previous figures, i.e. for many scales with sufficient density, a three-dimensional graph is constructed. It illustrates the dependence of $W_\Psi(a, \eta)$ and $W_\Psi(a, \phi)$ coefficients on η and ϕ, respectively, and scale a. The examples of the studied events are shown in Fig. 4.

Light colors correspond to maxima, dark colors to minima. An interpretation of the structures, i.e. the maxima and the minima, is the same as in the previous figures, only now we have much better resolution on the scale axes which allows more detailed study of the behaviour of the spectra in the scale domain.

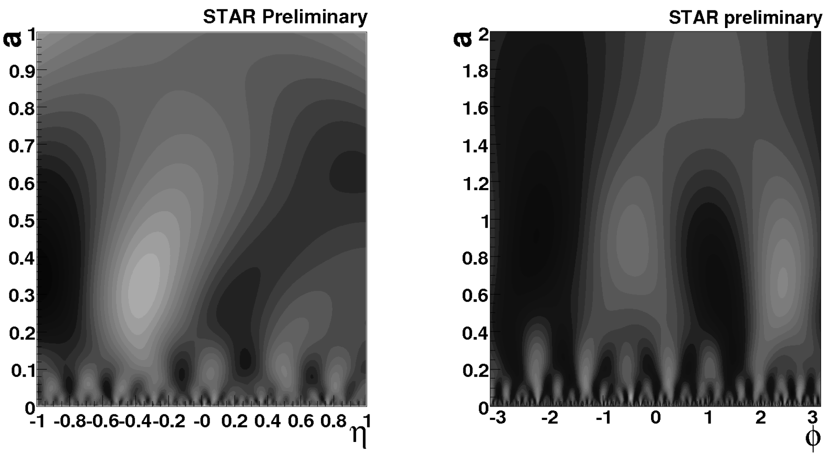


Fig. 4. Wavelet spectra of the η and ϕ distributions shown in Fig. 1.

3. Extreme points in the wavelet spectra

The previous example demonstrates that the local maxima in either the wavelet η or ϕ spectra suggest the characteristic scales and the prevailing pseudorapidities or azimuthal angles (in one single event). We may ask, are there any typical scales occurring systematically in many events or do they appear only randomly? It can be revealed by investigating extreme points in the wavelet η and ϕ spectra of a large sample of events. Motivation behind this question is that the occurrence of distinguished scales could hint at multi-particle flows produced by different physical mechanisms. We plan to employ the method to search for the collective flows induced by the production of hypothetic Cherenkov gluons [9] or Mach cones [10].

The local maxima found in the wavelet η and ϕ spectra presented in Fig. 4 are shown in Fig. 5. They indicate the presence of particle groups along with their η (or ϕ) and size, therefore they could be useful to map correlation structure of the studied events. Additionally, it is more comfortable to work with only a few significant points than with the whole wavelet spectra. We can investigate the number of wavelet maxima in individual events as well as their distribution in η-a (or ϕ-a) space.

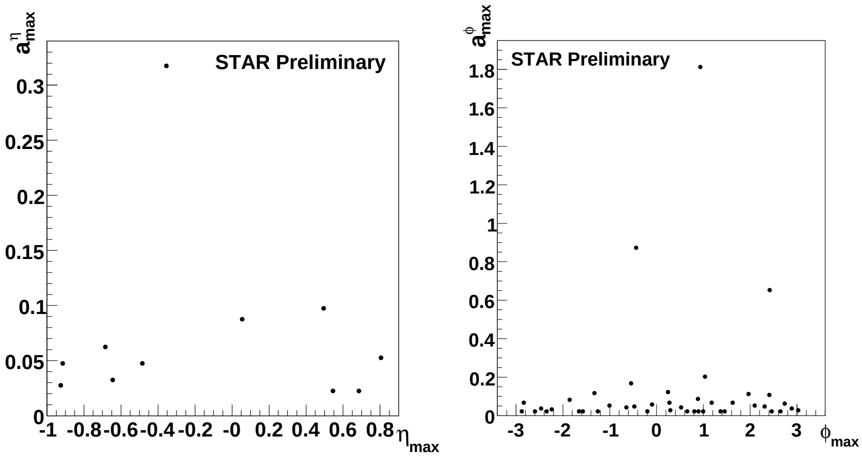


Fig. 5. Local maxima found in the wavelet η and ϕ spectra shown in Fig. 4.

The projections of wavelet maxima on scale axis may suggest significant scales. This can be alternatively achieved through the so-called scalogram defined as: $E(a) = \int W_\Psi(a,b)^2 \mathrm{d}b$ where b represents either η or ϕ. Its local maxima or bumps indicate relevant scales. Such spectra can be employed to sort events to various classes.

Furthermore, the projections of wavelet maxima on the η or ϕ axes with respect to some distinguished direction may imply prevailing pseudorapidities or azimuthal angles. The examples of distinguished directions are reaction plane in the ϕ case or a direction of motion of particle triggering Mach cone or Cherenkov gluon emission with some characteristic opening angle or pseudorapidity, if these are measured in the reference frame attached to the particle.

4. *Summary*

1) The continuous wavelet method is tentatively used to probe the angular distributions of secondary relativistic particles produced in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV.

2) The wavelet η and ϕ spectra of the small testing event sample show irregular structure of the angular spectra on all the studied scales. Local maxima in the wavelet spectra are associated with the groups of secondary particles and

imply their preferred emission angles as well as characteristic scales of the analyzed events. When performed on sufficiently large data samples, their thorough analysis may reveal the existence of particle collective effects of different nature.

3) Though the first preliminary tests prove the capability of the method to find and study multi-particle correlations, some improvements need to be implemented. They include the corrections for limited acceptance and efficiency of the STAR detector. Also a testing of the method on the Monte Carlo distributions with various shapes will be helpful for better understanding and interpretation of the real experimental data.

4) It will be necessary to develop and apply the wavelet based two-dimensional method providing simultaneous access to η and ϕ variables.

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References

[1] For reviews and recent developments see *Quark Matter 2006 Conference*, Yu-Gang Ma et al., J. Phys G: Nucl. Part. Phys. **34** (2006).

[2] K. H. Ackermann, Nucl. Instr. Met. in Phys. Res. A **499** (2003) 624.

[3] M. Anderson et al., Nucl. Instrum. Meth. A **499** (2003) 659.

[4] V. V Uzhinskii et al., Phys. Atom. Nucl. C **67** (2004) 156.

[5] J. Fedorišin, S. Vokál, E1-2007-4, JINR Dubna (2007).

[6] J. Fedorišin, S. Vokál, E1-2007-66, JINR Dubna (2007).

[7] M. Kopytine et al., Phys. Rev. C **71** (2005) 031901.

[8] I. M. Dremin, Phys. Usp. **43** (2000) 1137.

[9] I. M. Dremin, Nucl. Phys. A **767** (2006) 233.

[10] J. Casalderrey-Solana, E. V. Shuryak, D. Teaney, Nucl. Phys. A **774** (2006) 577.

ANALIZA VALIĆA VIŠEČESTIČNIH KORELACIJA

Proučavamo primjenjivost analize neprekidnih valića na pseudorapiditet i azimutalne raspodjele sekundarnih čestica proizvedenih u sudarima Au + Au na $\sqrt{s_{NN}} =$ 200 GeV u relativističkom teškoionskom sudaraču. Nakon popravaka zbog graničnih učinaka, valići se predstavljaju na različitim ljestvicama. Raspravljamo mogu li se preko ekstremnih vrijednosti u spektrima valića ili odgovarajućim skalogramima opaziti višečestični skupni tokovi proizvedeni različitim mehanizmima, te koji se daljni podaci o korelacijskoj strukturi mogu izvesti tom analizom.