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NONLINEAR WAVES IN A MAGNETIZED PLASMA – A DIFFERENT APPROACH

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An analysis of nonlinear waves in a low-frequency low-β magnetized plasma is performed in a modified approach. Two different nonlinear equations are deduced, having both solitary-wave-type and kink-type solutions. The solitary wave propagates obliquely to a static magnetic field. It is observed that both the current and magnetic field show the form of solitary and kink structure. Physical relevance of such nonlinear excitations is discussed.

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1. Introduction

A plasma is a medium which sustains various nonlinear phenomena, but each of them is observable on a different scale. It was due to this fact that the reductive perturbation technique of Washimi and Taniuti [1] and Roy Chowdhury et al. [2] proved to be so successful. The richness of the types of events can be attributed to the fact that plasma is a very nonlinear medium. On the other hand, without scaling the space and time, one can also scale the frequency, i.e., each physical variable (density, pressure etc.) can be separated into high- and low-frequency part and deduce the nonlinear equation. This was the approach followed in Rao et al. [3], Sharma et al. [4] and Shukla [5]. In the present communication, we wish to present an approach which does not rely on either of these methodologies, but rests on the basic equations almost in an exact manner, of course, under some assumptions. One of the main assumptions is that the z-component of the magnetic field B_z is constant inside the plasma. Under such an assumption, we can define two scalar potentials ϕ and ψ (Kadomstev [6]) and subsequently deduce the nonlinear equations for them. The solutions of these equations are then analysed and presented graphically.

2. *Low-β plasma*

We consider an electron-ion plasma placed in an external magnetic field. Furthermore, we assume that the usual hydrodynamic description is valid. Of course, our main concern is the region of low frequency (that is characteristic frequencies much lower than the ion cyclotron frequency) events along with low value of β. Here, β represents the ratio of the plasma pressure to the magnetic field pressure. It is given by

$$\beta = 2\mu_0 K_B (T_i + T_e) n_0 / B_0^2$$

The symbols we use, $m_e, m_i, T_e, T_i, \mu_0, K_B$ and n_0 are, respectively, the electron mass, ion mass, electron temperature, ion temperature, vacuum permeability, Boltzmann constant and equillibrium value of electron and ion density, n_e and n_i. Furthermore, we assume that ions are cold ($T_i = 0$) and there is a constant background magnetic field in the z-direction, $\vec{B} = \hat{z} B_0$. The collision-free equations describing the plasma are then written as:

$$0 = -e n_e (\vec{E} + \vec{v}_e \times \vec{B}) - \nabla (n_e K_B T_e), \tag{1}$$

$$m_i n_i \left[\frac{\partial \vec{v}_i}{\partial t} + (\vec{v}_i \cdot \nabla) \vec{v}_i \right] = e n_i (\vec{E} + \vec{v}_i \times \vec{B}), \tag{2}$$

$$\frac{\partial n_e}{\partial t} = -\nabla \cdot (n_e \vec{v}_e), \tag{3}$$

$$\frac{\partial n_i}{\partial t} = -\nabla \cdot (n_i \vec{v}_i), \tag{4}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \tag{5}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t}, \tag{6}$$

$$\nabla \cdot \vec{B} = 0, \tag{7}$$

$$\nabla \cdot \vec{E} = \frac{e}{\varepsilon_0} (n_i - n_e). \tag{8}$$

The current density $\vec{J}$ is defined to be equal to $e(n_i\vec{v}_i - n_e\vec{v}_e)$ and the magnetic field $\vec{B}$ is the sum of wave magnetic field and $\vec{B}_0$. Another important assumption is that there is no variation with respect to the y-coordinate.

For low frequency phenomena, $(1/g)\partial g/\partial t << \omega_{ci}$, where g is any quantity and $\omega_{ci} = eB_0/m_i$. It is due to this fact that the displacement current can be neglected, and from the beginning, we have neglected electron inertia [7]. We now introduce ϕ and $\vec{A}$ by

$$\vec{B} = \nabla \times \vec{A} \qquad \text{and} \qquad \vec{E} = -\frac{\partial \vec{A}}{\partial t} - \nabla\phi. \tag{9}$$

Because of the low-β assumption, we shall seek the configuration for which B_z is constant inside the plasma [8]. From Eq. (9), we get

$$A_y = B_{0x}, \qquad E_y = 0 \qquad \text{and} \qquad B_x = 0\,.$$

On the other hand, for the constant magnetic field configuration, we can always choose $A_y = 0$ as the gauge condition. These conditions show that we can define ψ via:

$$\psi = \phi + \int \frac{\partial A_z}{\partial t}\, \mathrm{d}z, \tag{10}$$

so that one gets at once

$$E_x = -\frac{\partial \phi}{\partial x}, \qquad E_z = -\frac{\partial \psi}{\partial z}. \tag{11}$$

From the y-component of Eq. (1), we get $v_{ex} = 0$ and the z-component of the same equation yields

$$n_e = n_{e0} \exp(\frac{e\psi}{K_B T_e}). \tag{12}$$

It is important to note that, since we are considering low-frequency Alfvén type excitation, the ion velocities in the transverse direction will be governed by the polarization drift and $\vec{E} \times \vec{B}$ drift. On the other hand, these can also be obtained by solving the linearized version of Eq. (2), which yields

$$v_{ix} = \frac{1}{B_0\omega_{ci}}\frac{\partial E_x}{\partial t} \qquad \text{and} \qquad v_{iy} = -\frac{E_x}{B_0}. \tag{13}$$

This is quite consistent with the fact that in a low-β plasma, the ion motion is primarily in the perpendicular direction. From Eq. (4) and using Eq. (13), we can derive

$$\frac{\partial n_i}{\partial t} = -\frac{1}{B_0\omega_{ci}}\left(n_i \frac{\partial^2 E_x}{\partial t \partial x} + \frac{\partial E_x}{\partial t}\frac{\partial n_i}{\partial x}\right), \tag{14}$$

but the Laplace equation leads to

$$n_i = n_{e0} \exp\left(\frac{e\psi}{K_B T_e}\right) - \frac{\varepsilon_0}{e}\left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\psi}{\partial z^2}\right). \tag{15}$$

Substituting (15) in (14), we get

$$\begin{aligned} &\frac{en_{e0}}{K_B T_e}\exp\left(\frac{e\psi}{K_B T_e}\right)\psi_t - \frac{\varepsilon_0}{e}(\phi_{xxt} + \psi_{zzt}) \\ &= \frac{1}{B_0\omega_{ci}}\left[\left\{n_{e0}\exp\left(\frac{e\psi}{K_B T_e}\right) - \frac{\varepsilon_0}{e}(\phi_{xx} + \psi_{zz})\right\}\phi_{xxt}\right. \\ &\left. + \phi_{xt}\left\{\frac{en_{e0}}{K_B T_e}\exp\left(\frac{e\psi}{K_B T_e}\right)\psi_x - \frac{\varepsilon_0}{e}(\phi_{xxx} + \psi_{xzz})\right\}\right], \end{aligned} \tag{16}$$

which is the first nonlinear equation for ϕ and ψ. To derive the other equation, we observe that the x-component of expression in (6) yields

$$\frac{\partial B_y}{\partial z} = -e\mu_0 v_{ix}, \tag{17}$$

and the y-component of expression in (9) gives

$$\frac{\partial B_y}{\partial t} = -\left(\frac{\partial^2\psi}{\partial x\partial z} - \frac{\partial^2\phi}{\partial x\partial z}\right). \tag{18}$$

Eliminating using these two equations and using the expression for v_{ix}, we get

$$\begin{aligned} &-\frac{e\mu_0}{B_0\omega_{ci}}\left[-\frac{\partial^2\phi}{\partial x\partial t}\left\{\frac{en_{e0}}{K_B T_e}\exp\left(\frac{e\psi}{K_B T_e}\right)\psi_t - \frac{\varepsilon_0}{e}\left(\frac{\partial^3\phi}{\partial x^2\partial t} + \frac{\partial^3\psi}{\partial z^2\partial t}\right)\right\}\right. \\ &\left. + \left\{n_{e0}\exp\left(\frac{e\psi}{K_B T_e}\right) - \frac{\varepsilon_0}{e}\left(\frac{\partial^2\phi}{\partial^2 x} + \frac{\partial^2\psi}{\partial^2 z}\right)\right\}\left(-\frac{\partial^3\phi}{\partial x\partial t^2}\right)\right] = -\psi_{xxz} + \phi_{xxz}. \end{aligned} \tag{19}$$

Equations (16) and (19) are the two coupled nonlinear partial differential equations for ϕ and ψ.

3. Dispersion relation

To get an idea of the nature of the wave sustained by the above coupled equations for ϕ and ψ, we linearize (16) and (19). The linearized forms of these equations are

$$\frac{en_{e0}}{K_B T_e}\psi_{1t} - \frac{\varepsilon_0}{e}(\phi_{1xxt} + \psi_{1zzt}) = \frac{n_{e0}}{B_0\omega_{ci}}\phi_{1xxt}, \tag{20}$$

$$\phi_{1xzz} - \psi_{1zzx} = \frac{e\mu_0 n_{e0}}{B_0 \omega_{ci}} \phi_{1xxt}. \tag{21}$$

If we now assume a propagating wave in the (x,z) plane to be of the form

$$\psi_1 = A_1 \mathrm{e}^{\mathrm{i}(k_x x + k_z z - \omega t)}$$

$$\phi_1 = A_2 \mathrm{e}^{\mathrm{i}(k_x x + k_z z - \omega t)}$$

then substituting in (20) and (21), we get the required dispersion relation

$$\omega^2 = V_A^2 k_z^2 \left[1 + \frac{k_x^2}{k_z^2} \left(\frac{\varepsilon_0 + 1/(\mu_0 V_A^2)}{\varepsilon_0 + 1/(k_z^2 \gamma_{Li}^2 \mu_0 V_A^2)} \right) \right]. \tag{22}$$

In Eq. (22), $\left(\mu_0 V_A^2\right)^{-1} >> \varepsilon_0 (\approx 1)$, hence this equation leads to

$$\omega^2 \approx V_A^2 k_z^2 \left[1 + \frac{k_x^2}{k_z^2} \frac{1/(\mu_0 V_A^2)}{1/(k_z^2 \gamma_{Li}^2 \mu_0 V_A^2)} \right] = V_A^2 k_z^2 \left(1 + \frac{C_s^2}{\omega_{ci}^2} k_x^2 \right),$$

which is the usual dispersion relation for the linear kinetic Alfvén waves. It includes the effect of finite gyroradius giving rise to the dispersion which competes with dispersion to generate the solitary wave. In these equations, $\gamma_{Li} = C_s/\omega_{ci} =$ ion gyroradius, $C_s =$ ion sound speed $= \sqrt{K_B T_e/m_i}$ and $V_A = \sqrt{B_0/(\mu_0 n_{e0} m_i)}$. This dispersion relation can be written also in the following form

$$k_z = \pm \sqrt{\frac{-(1-\omega^2\gamma_{Li}^2) \pm (1-\omega^2\gamma_{Li}^2)^2 + 4\omega^2\gamma_{Li}^2(\mathrm{cosec}^2\theta + \cot^2(\theta/V_A^2)}{2\gamma_{Li}^2 \cot^2\theta (1 + V_A^2 \sec^2\theta)}}, \tag{23}$$

where $\tan\theta = k_z/k_x$.

4. Solution of the nonlinear wave equation

Instead of considering the linearised form of the coupled system (16) and (19), let us now consider their full nonlinear version. We consider an arbitrary wave form

$$\phi = f(\lambda x + \sigma z - \omega t) = f(\xi), \tag{24}$$

$$\psi = g(\lambda x + \sigma z - \omega t) = g(\xi)$$

as a solution of Eqs. (16) and (19). This set is converted into the following nonlinear ordinary differential equations

$$\left(1 - \frac{\lambda^2 f''}{e n_{e0} \mu_0 V_A^2} \right) \left[\varepsilon_0 \left(\lambda^2 f'' + \sigma^2 g'' \right) - e n_{e0} \mathrm{e}^{\alpha g} \right] = 0, \tag{25}$$

$$f'' \left[\varepsilon_0 \left(\lambda^2 f'' + \sigma^2 g''\right) - e n_{e0} \mathrm{e}^{\alpha g}\right] = \frac{e n_{e0} V_A^2 \sigma^2}{\omega^2} (g'' - f''), \tag{26}$$

where $\alpha = e/(K_B T_e)$.

4.1. Case I

To proceed with the solution, we assume $1 - \lambda^2 f''/(e n_{e0} \mu_0 V_A^2) \neq 0$, so that $g'' = f''$, whence

$$g'' = \frac{e n_{e0}}{\varepsilon_0 (\lambda^2 + \sigma^2)} \mathrm{e}^{\alpha g}, \tag{27}$$

which is nothing but the Liouville's equation. One can at once write down two solutions

$$g_1 = \frac{1}{\alpha} \left[\log\left\{\frac{\alpha d (\lambda^2 + \sigma^2) \varepsilon_0}{2 e n_{e0}} \sec^2\left(\frac{\alpha\sqrt{d}}{2}(\xi + e')\right)\right\}\right], \tag{28}$$

$$g_2 = \frac{1}{\alpha}\left[\log(4\theta) - \log\{\mathrm{sech}^2(\delta + \alpha\xi/2)\}\right], \tag{29}$$

where $\theta = e n_{e0}/(2\varepsilon_0 \alpha(\lambda^2 + \sigma^2))$, δ and e' are arbitrary constants.

4.2. Case II

If $1 - \lambda^2 f''/(e n_{e0} \mu_0 V_A^2) = 0$, then it turns out that

$$f = \frac{e n_{e0} \mu_0 V_A^2}{2\lambda} \xi + D\xi + E \tag{30}$$

and

$$g = a_2 \sinh\left[\frac{\xi\sqrt{b/2}}{\sqrt{\sigma_1}} + \log\left(\frac{1}{a_2}\right)\right] - \frac{2a}{b}, \tag{31}$$

where $a_2^2 = (2/b)(c - 4a^2/b^2)$, $b = e n_{e0} \alpha$,

$$a = e n_{e0} - \left(\frac{e n_{e0} V_A^2 \sigma^2}{\omega^2} + \varepsilon_0 e n_{e0} \mu_0 V_A^2\right),$$

$$\sigma_1 = \varepsilon_0 \sigma^2 - \frac{e n_{e0} V_A^2 \sigma^2}{Q_1^2 \omega^2},$$

$$Q_1 = \frac{e n_{e0} \mu_0 V_A^2}{\lambda^2}.$$

We next consider the current density inside the plasma which can be computed from

$$\vec{J} = e(n_i \vec{v}_i - n_e \vec{v}_e). \tag{32}$$

Here we obtain the following expressions for the components of the current density in the Case I and Case II:

Case I.

$$J_x = 0 \qquad \text{if} \qquad g = f = g_1, \tag{33a}$$

$$J_x = \frac{\varepsilon_0 \alpha^2 \lambda \omega}{4B_0 \omega_{ci}} (\lambda^2 + \sigma^2) \left[\left(\frac{2en_{e0}}{\varepsilon_0 \alpha (\lambda^2 + \sigma^2)} \right)^2 - \mathrm{sech}^4 (\delta + \frac{\alpha \xi}{2}) \right], \quad \text{if} \quad g = f = g_2, \tag{33b}$$

$$J_z = -\frac{\omega e n_{e0}}{\sigma} \mathrm{e}^{\alpha\psi} \qquad \text{if} \qquad g = f = g_1, \tag{33c}$$

$$J_z = -\frac{\omega e n_{e0}}{\sigma} \mathrm{e}^{\alpha\psi} \left[1 - \frac{1}{2} \mathrm{sech}^2 (\delta + \frac{\alpha\xi}{2}) \right] \qquad \text{if} \qquad g = f = g_2\,. \tag{33d}$$

Case II.

$$J_x = \frac{\omega}{\lambda} \left[e n_{e0} (1 + \alpha \psi) - \varepsilon_0 \left\{ 2D_1 + \frac{b\sigma^2}{2\sigma_1} \left(\psi + \frac{2a}{b} \right) \right\} \right], \tag{34a}$$

$$J_z = -\frac{\omega e n_{e0} \alpha \omega}{\sigma} \left[\left(1 - \frac{2a\alpha}{b} \right) \left(\psi + \frac{2a}{b} \right) + \frac{\alpha}{4} \left\{ a_2^4 + 2 \left(\psi + \frac{2a}{b} \right)^2 \right\} \right]. \tag{34b}$$

Here, $D_1 = Q_1 \lambda^2 / 2$.

We can also calculate the magnetic field inside the plasma, which is actually one of the most important experimentally observed quantities. For Case I and Case II, we have

Case I.

$$B_y = -\frac{2\omega\lambda}{3\alpha\sigma V_A^2 \sqrt{d}} \left(\frac{\alpha d}{2} - \frac{e n_{e0}}{\varepsilon_0 (\lambda^2 + \sigma^2)} \right) \tan \left(\frac{\alpha\sqrt{d}}{2} (\xi + e') \right)$$

$$\times \left[2 + \sec^2 \left(\frac{\alpha\sqrt{d}}{2} (\xi + e') \right) \right] \qquad \text{for} \qquad g = f = g_1 \tag{35a}$$

$$B_y = -\frac{\lambda\omega\alpha}{V_A^2}\left[\frac{en_{e0}\sigma}{\varepsilon_0\alpha(\lambda^2+\sigma^2)} - \frac{\varepsilon_0(\lambda^2+\sigma^2)}{6en_{e0}\sigma}\tanh(\delta+\alpha\xi/2)\left\{2+\operatorname{sech}^2(\delta+\alpha\xi/2)\right\}\right]$$

$$\text{for} \qquad g = f = g_2\,. \qquad (35b)$$

Case II.

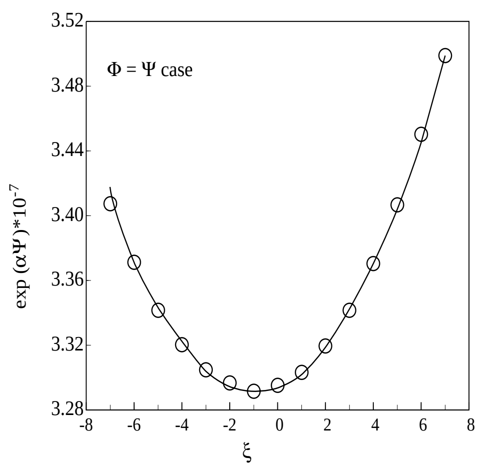
$$B_y = -\frac{en_{e0}\mu_0\omega}{\lambda} \times \left[\left(1-\frac{2a\alpha}{b}-\varepsilon_0\mu_0V_A^2\right)\sigma\right.$$

$$\left. + \frac{a_2\alpha}{\sigma}\left\{1-\left(2-\frac{2\lambda^2}{\varepsilon_0\mu_0\omega^2}\right)^{-1}\right\}\left\{\frac{2\varepsilon_0\sigma^2}{en_{e0}\sigma^2}\left(1-\frac{\lambda^2}{\varepsilon_0\mu_0\omega^2}\right)\right\}^{1/2}\right]$$

$$\times\cosh\left[\frac{\xi\sqrt{en_{e0}\alpha/2}}{\sqrt{\varepsilon_0\sigma^2(1-\lambda^2/(\varepsilon_0\mu_0\omega^2))}} + \log\left(\frac{1}{a_2}\right)\right]. \qquad (36)$$

5. Discussion

We now proceed to analyse the nature of the solutions ϕ and ψ and that of current and magnetic fields whose analytic expressions have been obtained above. In Fig. 1, we show the variation of $\exp(\alpha\psi)$ versus ξ for the case $\phi = \psi$, whereas



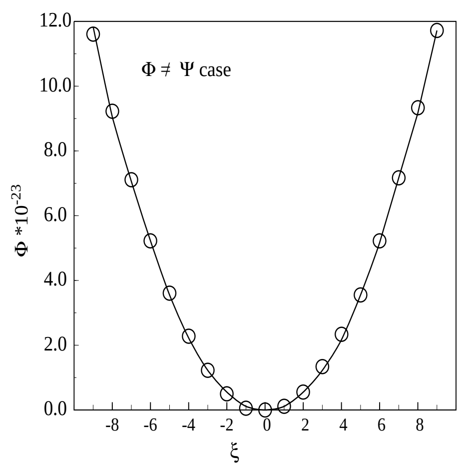
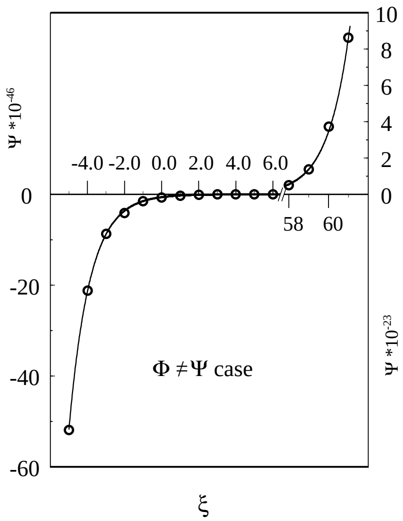


Fig. 1. The variation of $\exp(\alpha\psi)$ *with* ξ *when* $\phi = \psi = (1/\alpha)[\log\{\alpha d(\lambda^2+\sigma^2)\varepsilon_0\sec^2((\alpha/2)\sqrt{d}(\xi+e'))\}/(2n_{e0}] = g_1$.

Fig. 2 (right). Variation of ϕ *with* ξ*, for the* $\phi\neq\psi$ *case.*

Fig. 2 depicts that of ϕ versus ξ in the situation $\phi \neq \psi$. Both of these show and inverted solitonic profile whereas ψ for the later case is shown in Fig. 3 which clearly exhibits a kink profile. One may note the similarity with a shock-like structure [7].



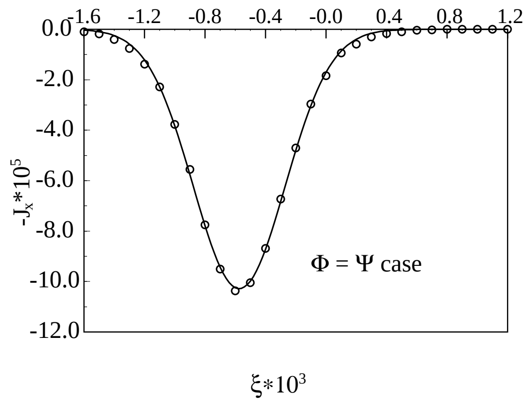
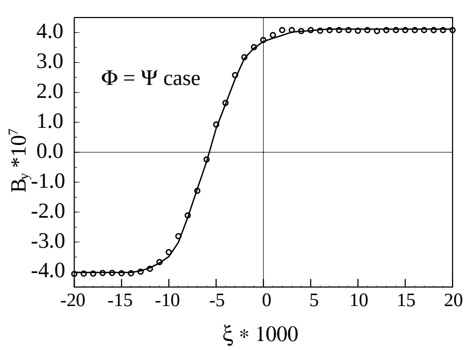


Fig. 3. Variation of ψ *with* ξ*, when* $\phi \neq \psi$.

Fig. 4 (right). Variation of the plasma current density (J_x) *with* ξ *when* $\phi = \psi = (1/\alpha)\ \ [\log\{2en_{e0}/(\varepsilon_0\alpha(\lambda^2+\sigma^2)\} - 2\log\mathrm{sech}(\delta+\alpha\xi/2)] = g_2$.



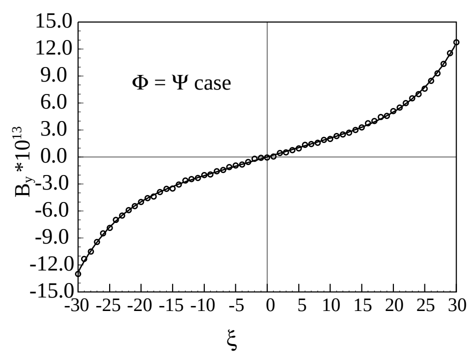
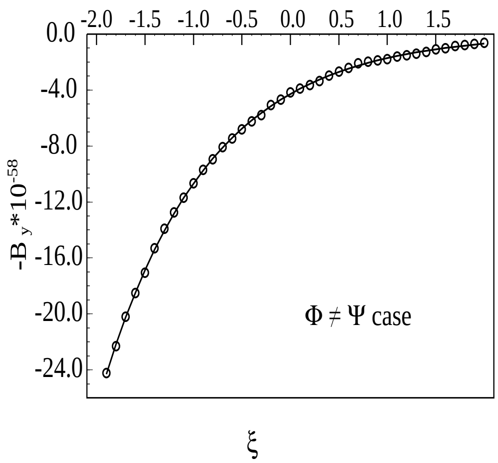


Fig. 5. Variation of the magnetic field (B_y) *with* ξ *when* $\phi = \psi = g_2$.

Fig. 6 (right). Variation of the magnetic field (B_y) *with* ξ *when* $\phi = \psi = g_1$.

So, here is an interesting physical situation, where we have both the solitonic and kink-type excitation simultaneously. Lastly, second solution of the $\phi = \psi$ case is exhibited in Fig. 4. To understand the situation better, we have calculated the induced magnetic field B_y and the current component J_z which we show in Figs. 5 to 9. While Fig 5. clearly depicts a kink-type structure for the component B_y for the first case of $\phi = \psi$. Fig. 6 shows a slightly different behaviour for the second case. On the other hand, in the situation $\phi \neq \psi$, the B_y component shows a saturation type tendency with ξ. Among the Figs. 8 and 9 only Fig. 8 shows a solitonic behaviour for the current component J_z. The situation for the other case is shown in the accompanying Fig. 9, which shows a quite different trend.



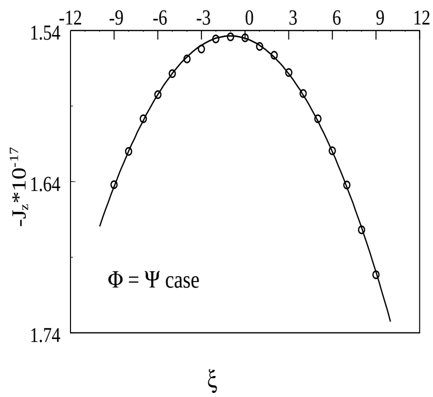


Fig. 7. Variation of the magnetic field (B_y) with ξ when $\phi \neq \psi$.

Fig. 8 (right). Variation of the current density (J_z) with ξ when $\phi = \psi = g_1$.

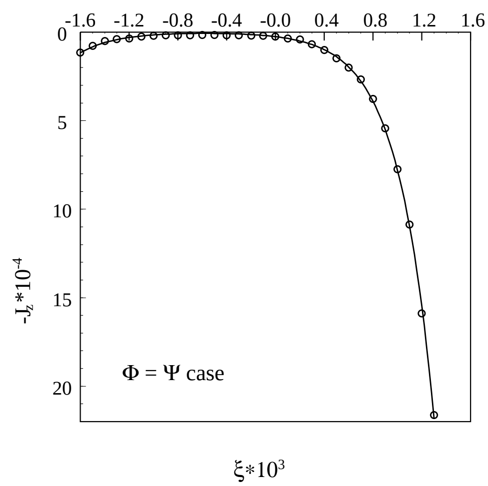


Fig. 9. Variation of the current density (J_z) with ξ when $\phi = \psi = g_2$.

The situation discussed above is interesting due to the fact that these excitations give rise to a D.C. current in the plasma in the steady state. Both the longitudinal and transverse components of the current exist and exhibit a behaviour similar to a nonlinear wave except for a particular situation. Actually, by a D.C. current we mean a time averaged current. In this connection, it may be mentioned that the possibility of producing a D.C. current by using a wave has been extensively considered in the last few years [8,9]. The motivation was that if such steady current can be generated in a tokamak [10], then it can be operated continuously. Lastly, we may add a few comments about the idea of D.C. current. From the basic law connecting J_z and B_y, we can define

$$J_z = \int J_z \mathrm{d}n,$$

whence $J_z = 2k_x B_y(y \to \infty)$. J_z is proportional to the total longitudinal current in a slab of unit width in y-direction. It is actually proportional to the total charge that passes through unit area in unit time. In general, the average of any wave-like quantity is zero, but if we think that we have a train of well separated solitons each following the other but not interacting, then it is possible to have a non-zero value. That is why we term such a J_z current as a D.C. one.

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NELINEARNI VALOVI U MAGNETIZIRANOJ PLAZMI – NOV PRISTUP

Načinili smo analizu nelinearnih valova u niskofrekventnoj magnetiziranoj plazmi niskog β u izmijenjenom pristupu. Izveli smo dvije različite nelinearne jednadžbe. Obje imaju solitonsko i skokovito rješenje. Solitonski se val širi koso u odnosu na smjer statičkog magnetskog polja. Opaža se da i struja i magnetsko polje pokazuju solitonsku i skokovitu strukturu. Raspravlja se fizičko značenje tih nelinearnih uzbuda.